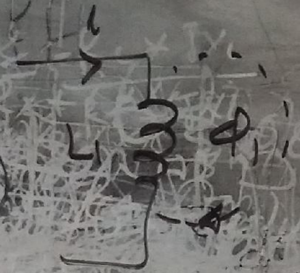
11

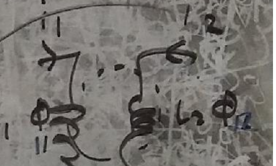
1 Self inductance



$$L = \frac{\mu A N^2}{L} = N\frac{d\Phi_1}{di_1} = \left(\frac{N\Phi_1}{I_1}\right)$$

$$v_{L_1} = L\frac{di}{dt} = N\frac{d\Phi}{dt}$$

2 Mutual inductance

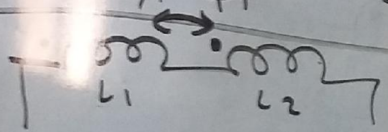


$$M = \frac{N_1\Phi_{21}}{I_2} = \frac{N_2\Phi_{12}}{I_1} = K\sqrt{L_1 L_2}$$

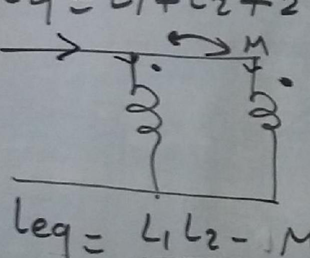
$$\Phi_1 = \Phi_{11} + \Phi_{12}$$ Total flux

coupling coeff. $K = \frac{\Phi_{12}}{\Phi_1} = \frac{\Phi_{21}}{\Phi_2}$

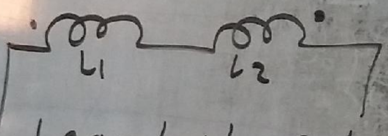
3



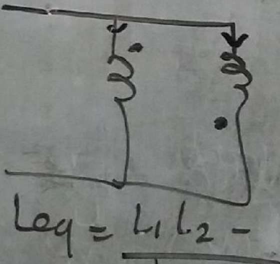
$$L_{eq} = L_1 + L_2 + 2M$$



$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$



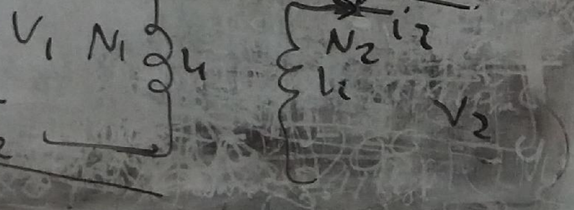
$$L_{eq} = L_1 + L_2 - 2M$$



$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

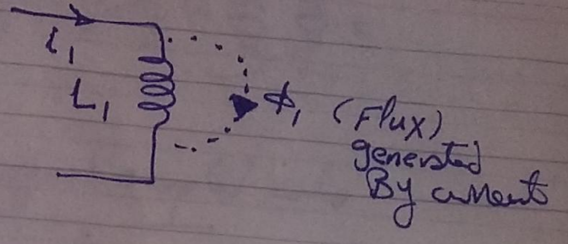
4 Transformer

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1} = \sqrt{\frac{L_1}{L_2}}$$



ملخص القوانين

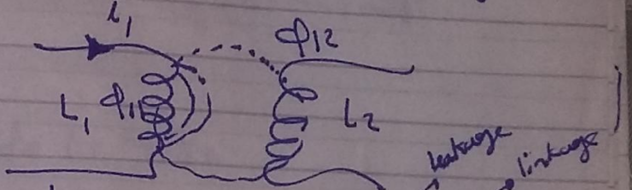
$\boxed{1}$ Self inductanc



(a) v_{L_1} (induced emf) $= N_1 \frac{d\phi_1}{dt}$
$= L_1 \frac{di_1}{dt}$

(b) $L_1 = N_1 \frac{d\phi_1}{di_1}$ → For linear sy $L_1 = N_1\phi_1/i_1$
→ for sinusoidal $U_{L_1} = L_1\frac{di_1}{dt} = j\omega L_1 i_1$

$\boxed{2}$ Mutual inductanc



a- $M = K\sqrt{L_1 L_2}$ henny

where b- $M = \underbrace{\frac{N_1 d\phi_{21}}{di_2}}_{M_{21}} = \underbrace{\frac{N_2 d\phi_{12}}{di_1}}_{M_{12}}$

$\phi_1 = \overset{\text{leakage}}{\phi_{11}} + \overset{\text{linkage}}{\phi_{12}}$
$\phi_2 = \phi_{22} + \phi_{21}$

c- $K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$ = Coupling coff ≤ 1
depend on
→ Space bet^n coils
→ μ of medium bet^n coils
→ orientation of axis

d- $\cancel{X}_m = K\sqrt{X_{L_1} X_{L_2}}$ Ω

$\boxed{3}$ self inductanc Parameters

$L = \frac{N^2 \mu A}{L}$ (henery)

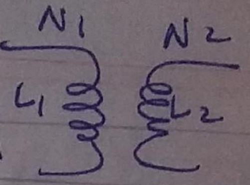
N → No: of Turns
L → length of coil
A → area

$\mu = \mu_o \mu_r$
μ_o → permeability of free space $4\pi \times 10^{-7}$
μ_r → preablty of ivarse

②

3) Transformer

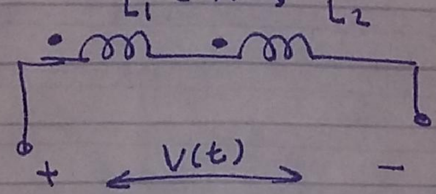
→ Turns Ratio $= \frac{N_1}{N_2} = \frac{N_p}{N_s} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$

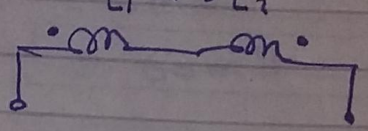


→ $\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}}$

Magnetic coupled coils

[4] sign of M (series/parallel) .. dot convention





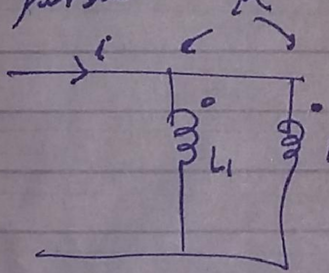
$L_{t_2} = L_1 + L_2 - 2M$

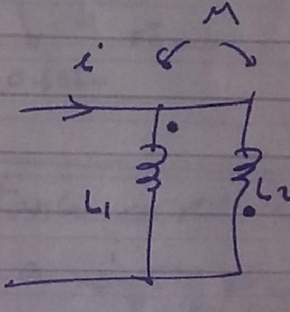
$L_{t_1} = L_1 + L_2 + 2M$

[illegible]

Note $L_{T_1} > L_{T_2}$

Parallel





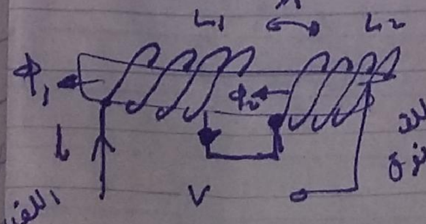
$L_{t_1} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$

$L_{T_2} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$

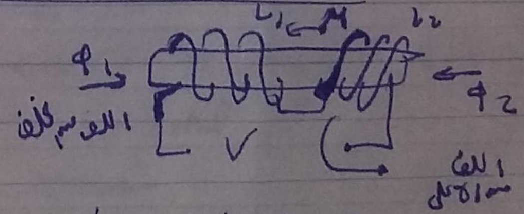
$L_{t_1} > L_{t_2}$

[illegible]

شكل آخر لل series



$L_{eq} = L_1 + L_2 + 2M$



$L_{eq} = L_1 + L_2 - 2M$

③

Sheet (6)

1) one coil of magnetically coupled pair has current 5A Φ_{11}, Φ_{12} are 0.2 mwb , 0.4 mwb Respectively If $N_1 = 500$, $N_2 = 1500$, Find L_1, L_2, M, K?

Sol

$\rightarrow \Phi_1 = \Phi_{11} + \Phi_{12} = 0.2 + 0.4 = 0.6\,mb$

$\rightarrow L_1 = \frac{N_1 \Phi_1}{I_1} = \frac{500(0.6)}{5} = 60\,mH$

$\rightarrow K = \frac{\Phi_{12}}{\Phi_1} = \frac{0.4}{0.6} = 0.6667$

$\rightarrow M = \frac{N_2 \Phi_{12}}{I_1} = \frac{1500(0.4)}{5} = 120\,mH$

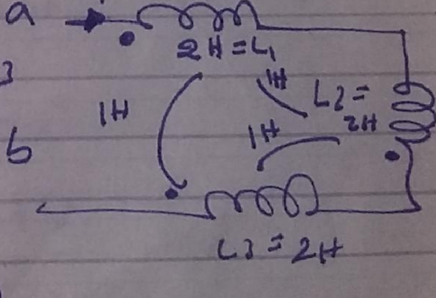
$\rightarrow K = \frac{M}{\sqrt{L_1 L_2}}$ or $M = K\sqrt{L_1 L_2}$

120mH 0.6667 60mH

$\therefore L_2 = 540\,mH$

---

2 find The equivalent inductance across terminals a, b!



$L_{eq} = L_1 + L_2 + L_3 - 2M_{12} + 2M_{23} - 2M_{13}$
$= 6 - 2 - 2 + 2 = 4H$

NB

$v = (jwL_1 i - jwM_{12} i - jwM_{13} i) +$
$(jwL_2 i - jwM_{12} i + jwM_{23} i) +$
$(jwL_3 i + jwM_{23} i - jwM_{13} i) = jwi [L_1 + L_2 - 2M_{12} -$
$2M_{13} + 2M_{23}) = jwL_{total}$

-[B]

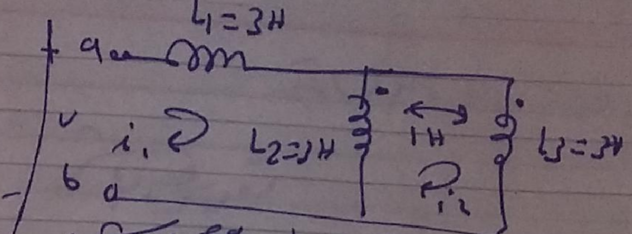
ط بالتيارات

بكرامر؟

Loop ①

$= j\omega L_1 i_1 + (i_1 - i_2) j\omega L_2 + j\omega M i_2$

$= i_1 (j\omega L_1 + j\omega L_2) + i_2 (j\omega M - j\omega L_2) \rightarrow ①$



∴ نجيب eq ل ط

$$L_{23} = \frac{L_2 L_3 - M^2}{L_2 + L_3 - 2M} = 2H$$

$$L_{eq} = L_1 + L_{23} = L_1 + L_{23} = 5H$$

Loop ②

$$0 = \left[\underline{j\omega L_3 i_2} - \underset{\text{العلامة بالعكس}}{j\omega M i_2} \right] + \left(\underline{(i_2 - i_1) j\omega L_2} - j\omega M (i_2 - i_1) \right)$$

$$0 = i_1 [j\omega M - j\omega L_2] + i_2 [j\omega L_3 - j\omega M + j\omega L_2 - j\omega M] \rightarrow ②$$

$$\Delta = \begin{bmatrix} j\omega(L_1 + L_2) & j\omega(M - L_2) \\ j\omega(M - L_2) & j\omega(L_2 + L_3 - 2M) \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} V & j\omega(M - L_2) \\ 0 & j\omega(L_2 + L_3 - 2M) \end{bmatrix}, \quad I_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta_2 = \begin{bmatrix} j\omega(L_1 + L_2) & V \\ j\omega(M - L_2) & 0 \end{bmatrix}, \quad I_2 = \Delta_2 / \Delta$$

اذا عوضنا ال معاملات

note $Z_{in} = Z_{ab} = \dfrac{\Delta}{\Delta_{11}} \rightarrow$ الغي الصف الاول العمود

Δ ل

$[\, 4 \quad ⓪ \,]$

$$Z_{in} = \frac{(j\omega)^2 20}{j\omega(4)} = j\omega(5)$$

∴ $L_{total} = 5\ H$

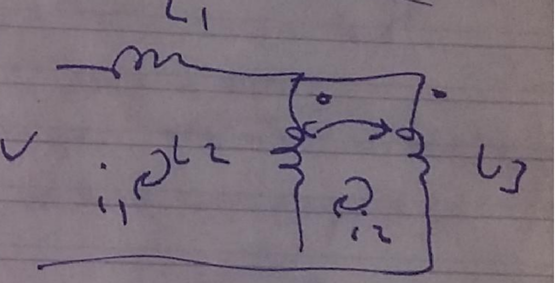
⑤

حل آخر للسابق

تعويض مباشر

$$L_{23} = \frac{L_2 L_3 - M^2}{L_2 + L_3 - 2M} = 2H$$

$$L_{eq} = L_1 + L_{23} = 5H$$

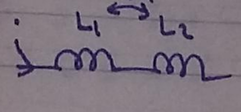


① $V = i_1 (X_{L1} + X_{L2}) -$
$i_2 X_{L2} + i_2 X_M$

②
~~$0 = i_2 (X_{L3} + X_{L2}) + i_1 X_M - M i_2 j\omega$~~
~~$- i_1 X_{L2}$~~

$0 = \cancel{i_2 (X_{L2} + X_{L3})} - \underline{(i_2 - i_1)} j\omega M + \underline{(i_2 - i_1)} X_{L2}$
$+ \underline{X_{L3} i_2} - j\omega M i_2 \longrightarrow ②$

3 – Calc. mutual inductance of 2 coils of self inductance 100mH, and 200mH are connected in series to yield a total inductance of 146mH.



sol/.

$$L_{eq} = L_1 + L_2 \pm 2M$$

$$146m = 100m + 200m \pm 2M$$

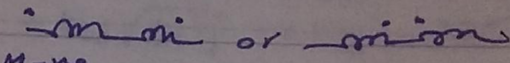
$$\therefore \pm 2M = (146 - 300)m \Rightarrow \pm M = \pm 77mH$$

$$\therefore \quad \therefore M = -77mH \text{ or } \underline{M = 77mH}$$

$\underline{\underline{M\ -ve}}$

لاحظ ان

$L_1 + L_2 > 146mH \therefore$

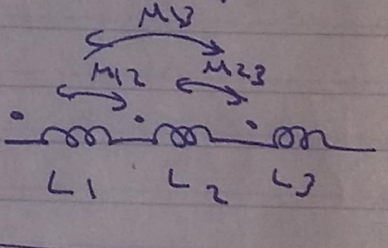


M -ve

---

4 3 similar coil wound on a long common core, The voltage of mutual inductance between each set of coil is +ve, The self inductance of each coil is 0.2H. The effective inductance of first 2 coils is 0.6H, and of all 3 in series is 1H. (When Terminals of first coil is interchanged, The effective inductance of 3 coils in series become 0.5H) determine coeff of coupling b/w each set of coil

sol



→ $L_1 = L_2 = L_3 = L = 0.2H$

→ $L_{12} = 0.6H$, $L_{total} = 1H$

→ $L_{12} = \underbrace{L_1}_{0.2} + \underbrace{L_2}_{0.2} + 2M_{12} = 0.6 \quad \therefore \boxed{M_{12} = 0.1H}$

→ $M_{12} = K_{12}\sqrt{L_1 L_2} = K_{12}\sqrt{0.2 \times 0.2} = K_{12} \times 0.2 \quad \therefore \boxed{K_{12} = 0.5}$

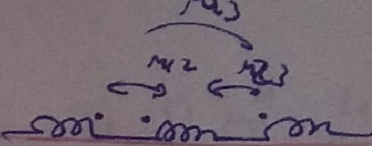
→ $L_{total} = L_1 + L_2 + L_3 + 2M_{12} + 2M_{23} + 2M_{13}$

$1 = 3 \times 0.2 + 2 \times 0.1 + 2M_{23} + 2M_{13} \quad \therefore \boxed{M_{13} + M_{23} = 0.1}$

↘ 1

~~3~~

When coil 1 غيرنا أطرافه



$\therefore$ M12, M13 → -ve

$\boxed{L_{T new} = 0.5}$

$0.5 = L_1 + L_2 + L_3 - 2M_{12} + 2M_{23} - 2M_{13}$

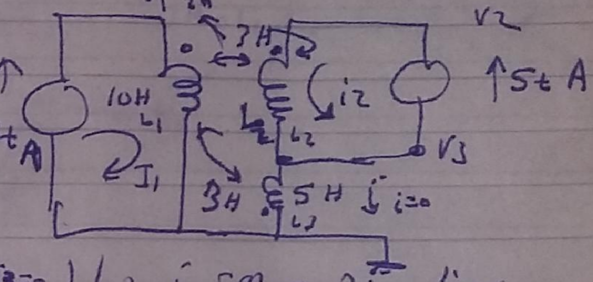
$0.6 = 0.6 - 0.2 + 2M_{23} - 2M_{13}$

$\therefore \boxed{M_{23} - M_{13} = 0.05} \rightarrow 2$

Solve 1, 2 $\therefore M_{13} = 0.025 = K_{13}\sqrt{L_1 L_3} \rightarrow K_{13} = 0.125$

$M_{23} = 0.075 = K_{23}\sqrt{L_2 L_3} \rightarrow K_{23} = 0.375$

$\boxed{5}$ find V_1, V_2, V_3

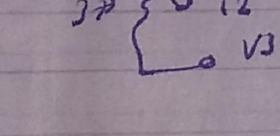


(I3=0) L3 لو مفيش تيار معدي ف
لذا مش هيكون فيه أثر ناتج
معامل L3 و ...
بس يهمني تأثير I1

$\rightarrow V_1 = L_1 \frac{dI_1}{dt} + M_{12}\frac{dI_2}{dt} + \cancel{M_{13}\frac{dI_3}{dt}}_{0}$

$= 10 \times 2 + 2 \times 5 = 30V$

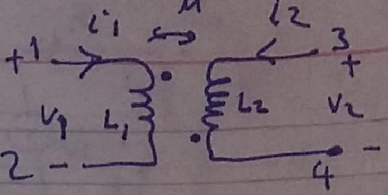
$\rightarrow V_3 = M_{13}\frac{dI_1}{dt} + L_3\frac{dI_3}{dt}_{\hookrightarrow 0} = 3 \times 2 = 6V$



$V_2 - V_3 = L_2\frac{di_2}{dt} + M_{21}\frac{di_1}{dt}$

$V_2 - V_3 = 3 \times 5 + 2 \times 2 = 19V$

$V_2 = V_3 + 19 = 25V$



$\boxed{5}$ → $L_2 = 2H$, The inductance at 1,2 is 3H when terminals 3, 4 opened

→ And 1H when shorted,

Determine coeff of coupled ckt.

M

1- $V_{12} = L_1 \frac{di_1}{dt} - M_{12} \frac{di_2}{dt}$

2- $V_{34} = L_2 \frac{di_2}{dt} - M_{21} \frac{di_1}{dt}$

for 3,4 (o.c) → $i_2 = 0$ $\therefore di_2 = 0$

$\therefore V_1 = L_1 \frac{di_1}{dt} = jwL_1 i_1 = jX_L \cdot i_1$

$\therefore \boxed{L_1 = 3H}$ $\because V_1$ دالتي لها بس $\therefore L_1 = 3H$

for 3,4 S.C $V_2 = 0$ only

$0 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$

$L_2 \frac{di_2}{dt} = M \frac{di_1}{dt}$

$\therefore \frac{di_2}{dt} = \frac{M}{L_2} \frac{di_1}{dt}$

$\therefore V_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = L_1 \frac{di_1}{dt} - \frac{M^2}{L_2} \frac{di_1}{dt} = (L_1 - \frac{M^2}{L_2}) \frac{di_1}{dt}$

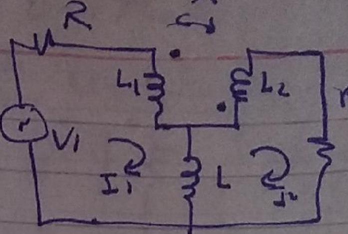
$L_{eq} = \underset{3}{L_1} - \frac{M^2}{L_2} = 1H$ $\therefore 3 - \frac{M^2}{\underset{2}{L_2}} = 1$

$\therefore \boxed{M = 2H}$

$M = K\sqrt{L_1 L_2}$ $\therefore K = 0.816$

$\boxed{7}$ ratio I_1/I_2 = ?! Using < directly coupled ckt / conductively equivalent ckt

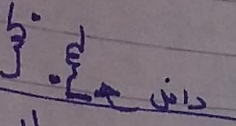
sol.



$\boxed{1}$ $V_1 = I_1(R + j\omega(L_1+L)) - i_2 j\omega L$

$\textcircled{+} j$ ~~4e~~ $\omega M i_2$

داخل



$\therefore$

$$\boxed{V_1 = I_1(R + j\omega(L+L_1)) + j\omega(M-L)\, i_2}$$

$\boxed{2}$

$$\boxed{0 = I_2(R + j\omega(L+L_2)) + j\omega(M-L) I_1}$$

$$\Delta = \begin{vmatrix} R+j\omega(L+L_1) & j\omega(M-L) \\ j\omega(M-L) & R+j\omega(L+L_2) \end{vmatrix}$$

$$= \boxed{V_1[R+j\omega(L+L_2)]}$$

$$\Delta_1 = \begin{vmatrix} V_1 & j\omega(M-L) \\ 0 & R+j\omega(L+L_2) \end{vmatrix}$$

$$= \boxed{V_1[R+j\omega(L+L_2)]}$$

$$\Delta_2 = \begin{vmatrix} R+j\omega(L+L_1) & V_1 \\ j\omega(M-L) & 0 \end{vmatrix}$$

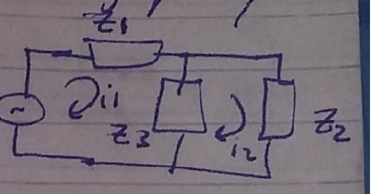
$$= -V_1(j\omega(M-L)) = \boxed{V_1(j\omega(L-M))}$$

$I_1 = \Delta 1/\Delta$ $\qquad I_2 = \Delta 2/\Delta$

$$I_1/I_2 = \Delta 1/\Delta 2 = \boxed{\frac{R+j\omega(L+L_2)}{j\omega(L-M)}}$$

نحولها إلى :

Conductively coupled equivalent



$V = (Z_1+Z_3) i_1 - Z_3 i_2 \rightarrow ③$

$0 = -Z_3 i_1 + (Z_2+Z_3) i_2 \rightarrow ④$

compare 3, 4 by 1, 2

$\therefore Z_1 + Z_3 = R + j\omega(L_1+L) \rightarrow ⑤$

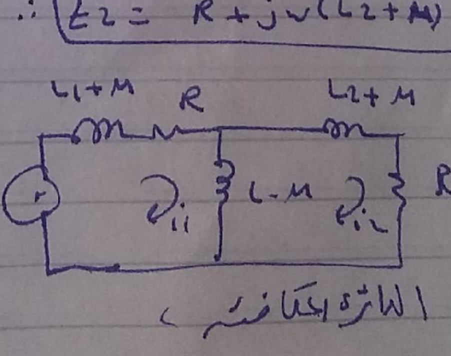
$-Z_3 = j\omega(M-L)$

or $\boxed{Z_3 = j\omega(L-M)} \rightarrow ⑥$

$\therefore \boxed{Z_1 = R + j\omega(L_1+M)} \rightarrow ⑦$

$Z_2 = R + j\omega(L_2+L) - j\omega(L-M)$

$\therefore \boxed{Z_2 = R + j\omega(L_2+M)} \rightarrow 8$



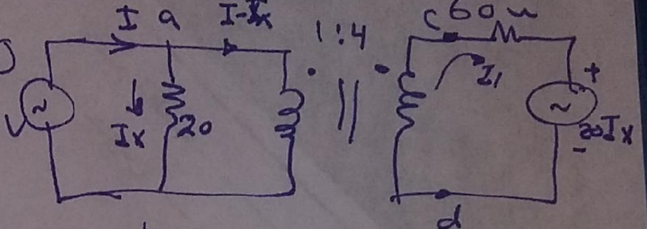
الدائرة المكافئة

فقرة زيادة

Find impedance faced by Voltage source V in the circuit

sol

From figure $V = V_{ab} = \overset{IR}{20 I_x} \rightarrow$ ①



→ Current Through Primary

$= I - I_x = I_p$

∴ $\frac{N_p}{N_s} = \frac{I_s}{I_p} = \frac{1}{4} = \frac{I_s}{I - I_x}$ → $I_s = \frac{I - I_x}{4} = I_1 \rightarrow$ ②

→ $\frac{N_p}{N_s} = \boxed{\frac{1}{4} = \frac{V_p}{V_s} = \frac{V}{V_{cd}}}$

$$\therefore V_{cd} = 4V = \underbrace{I_s \times 60}_{IR} + \underbrace{20 I_x}_{+V_{source}} = 60 * \frac{I - I_x}{4} + 20 I_x$$

$$\therefore 4V = 15(I - I_x) + 20 I_x$$

from ① $4V = 15I - \underbrace{15 I_x + 20 I_x}_{5 I_x}$

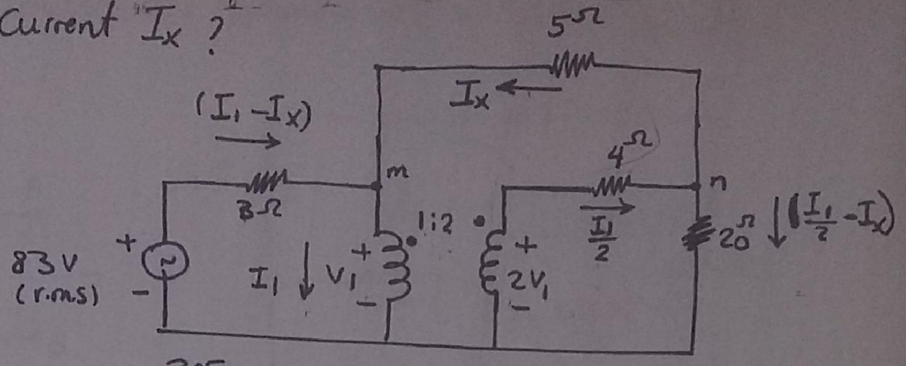
~~4(20Ix) =~~

$$4V = 15I + 5\left(\frac{V}{20}\right)$$

∴ $V = 4I$ ∴ $\frac{V}{I} = 4$ = impedance

Q.11)

Determine the Current I_x ?



answer.

- V_1 = primary voltage , I_1 = primary cu^t
- $\frac{N_1}{N_2} = \frac{N_P}{N_S} = \frac{1}{2} = \frac{V_P}{V_S} = \frac{I_S}{I_P}$
- $V_S = 2V_P = 2V_1$
- $I_S = \frac{I_P}{2} = \frac{I_1}{2}$
- KCL at m ⇒ I(on 3Ω resis) $= I_1 - I_x$
- KCL at n ⇒ I(on 20Ω resi) $= \frac{I_1}{2} - I_x$.

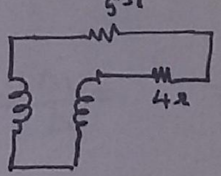
- KVL at primary loop:

$$83 = 3(I_1 - I_x) + V_1 \rightarrow ①$$

- KVL at Secondary loop:

$$2V_1 = 4\left(\frac{I_1}{2}\right) + 20\left(\frac{I_1}{2} - I_x\right) \rightarrow ②$$

- apply the KVL for the loop shown



$$2V_1 = 4 \cdot \frac{I_1}{2} + 5I_x + V_1$$

$$\downarrow$$

$$\therefore V_1 = 2I_1 + 5I_x \rightarrow ③$$

عوض بـ ③ في ② ، ① :

$$\therefore 83 = 3(I_1 - I_x) + (2I_1 + 5I_x) = 5I_1 + 2I_x \rightarrow ④$$

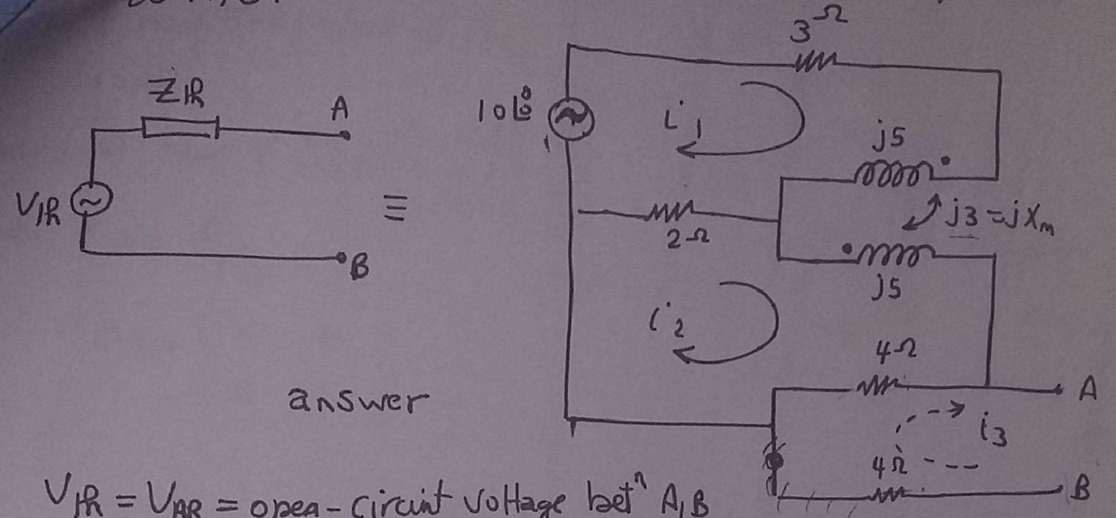
$$\& \quad 2(2I_1 + 5I_x) = 4\left(\frac{I_1}{2}\right) + \frac{20}{2}I_1 - \frac{20}{2}I_x$$

$$\therefore 8I_1 = 30I_x \rightarrow ⑤$$, solve ④ ⑤

$$\therefore \boxed{I_x = 4A}$$ r.m.s

$X_m = \cancel{k}\sqrt{X_1 \cdot X_2}$

Q.12) Replace the Coupled net^k shown with a Thevenin equivalent to A, B:



$V_{th} = V_{AB}$ = open-circuit voltage bet^n A, B

$= I_2\,(4^\Omega)$ → the other (4^Ω) resistor has no voltage drop on it (open).

??

we need to write the mesh equations of the (2-loop) ckt:

$$\text{loop } \boxed{1} \Rightarrow 10\angle 0^\circ = i_1(3^\Omega) + 2^\Omega(i_1 - i_2) + [j5(i_1) + \overset{X_m}{j3}\, i_2] \rightarrow ①$$

$$\text{loop } \boxed{2} \Rightarrow 0 = 2(i_2 - i_1) + 4(i_2) + [j5(i_2) + j3\, I_1] \rightarrow ②$$

$$\therefore \Delta = \begin{bmatrix} 5+j5 & -2+j3 \\ -2+j3 & 6+j5 \end{bmatrix}$$

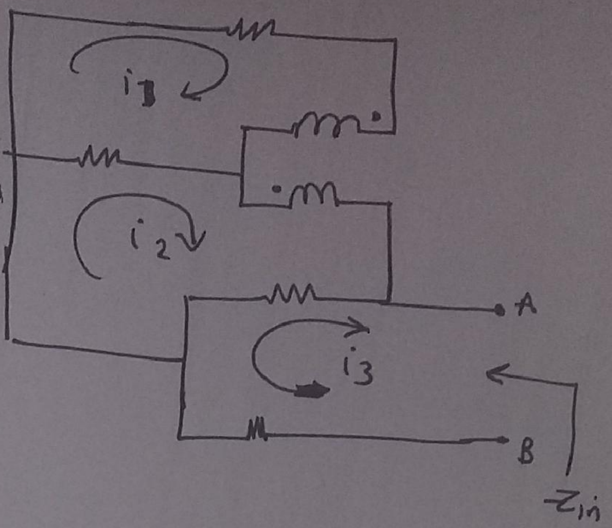
$$\Delta_2 = \begin{bmatrix} 5+j5 & 10\angle 0^\circ \\ -2+j3 & 0 \end{bmatrix} \Rightarrow I_2 = \frac{\Delta_2}{\Delta} = \quad = 0.533\angle -137.8^\circ \text{ A}$$

$$\therefore V_{th} = [0.533\angle -137.8^\circ] * 4^\Omega = 2.13\angle -137.8^\circ$$

⇒ To find Z_{th} ? 1) S.C all voltage sources.
2) O.C all cu^t sources.
3) find Z_{in} bet^n A, B

we have now (3-loops) when looking from A, B as follow:

⇒ we need 3-equations of the loops:



$\Rightarrow$ find new $(\grave{\Delta}) \equiv \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$\Rightarrow Z_{in} = \dfrac{\grave{\Delta}}{\Delta_{33}}$

$\Delta_{33} = \begin{bmatrix} a & b \\ d & e \end{bmatrix}$

loop ① ⇒ $0 = i_1(3) + 2(i_1 - i_2) + [\underset{\text{self}}{j5\, i_1} + \underset{\text{mut}}{j3\, i_2}] \longrightarrow$

$0 = i_1(5+j5) + i_2(-2+j3) + i_3(0) \rightarrow ①$

Loop ② ⇒ $0 = 2(i_2 - i_1) + 4(i_2 - i_3) + [\underset{\text{self}}{j5\,(i_2)} + \underset{\text{mut.}}{j3\, I_1}]$

$0 = i_1(-2+j3) + i_2(6+j5) + i_3(-4) \rightarrow ②$

Loop ③ $0 = 4(i_3 - i_2) + 4(i_3)$

$0 = i_1(0) + i_2(-4) + i_3(8) \rightarrow ③$

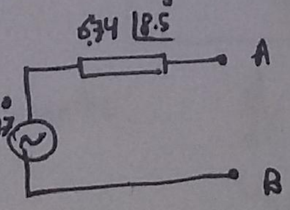
$$\Delta' = \begin{bmatrix} 5+j5 & -2+j3 & 0 \\ -2+j3 & 6+j5 & -4 \\ 0 & -4 & 8 \end{bmatrix} = \checkmark$$

$$\Delta_{33} = \begin{bmatrix} 5+j5 & -2+j3 \\ -2+j3 & 6+j5 \end{bmatrix} = \checkmark$$

$$Z_{th} = Z_{in} = Z_{AB} = \frac{\Delta'}{\Delta_{33}} = \qquad = \underline{6.74\angle 8.5^\circ\ \Omega}$$

$= 6.65 + j\,0.996$

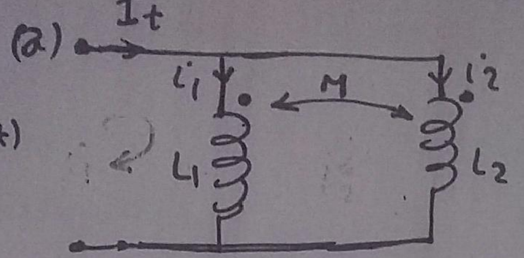
∴



— [illegible]

⇨ For parallel Connections:

(a)



$$v(t) = L_1 \cdot \frac{di_1}{dt} + M\frac{di_2}{dt}$$

&

$$v(t) = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

or sinus.

$$j\omega L_1 i_1 + j\omega M i_2 = v.$$

$$j\omega M I_1 + j\omega L_2 I_2 = V$$

$$\Delta = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix}$$

$$= \omega^2(M^2 - L_1 \cdot L_2)$$

$$\Delta_1 = \begin{bmatrix} V & j\omega M \\ V & j\omega L_2 \end{bmatrix} = V(j\omega)(L_2 - M)$$

$$\Delta_2 = \begin{bmatrix} j\omega L_1 & V \\ j\omega M & V \end{bmatrix} = V(j\omega)(L_1 - M)$$

$$I_t = I_1 + I_2 = \frac{\Delta_1}{\Delta} + \frac{\Delta_2}{\Delta}$$

$$I_t = \frac{j\cancel{\omega}(L_1 + L_2 - 2M)V}{\omega^{\cancel{2}}(M^2 - L_1L_2)}$$

$$\frac{V}{I_t} = Z_{in} = \frac{\omega(M^2 - L_1L_2)}{j(L_1 + L_2 - 2M)}$$

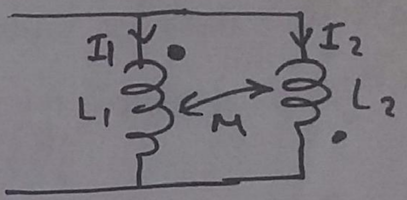
$$\therefore Z_{in} = \frac{\omega(M^2 - L_1L_2)}{j(L_1 + L_2 - 2M)} \cdot \frac{j}{j}$$

$$= j\omega\left[\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right]$$

$$= j\omega L_{eq} = j\omega L_a$$

$$\therefore \boxed{L_a = \frac{L_1 \cdot L_2 - M^2}{L_1 + L_2 - 2M}}$$

(b)



بالمثل

$$\boxed{L_{eq} = L_b = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}}$$

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